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Interphase Boundary Waves in the Intermediate State of Lead†

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The propagation of waves similar to helicon waves has been studied in the superconducting intermediate state of lead in polycrystalline samples and in single crystals of varying quality. The theories of Nozières and Vinen and of Andreev were found to agree well with the results from the highest-quality specimen. The behavior of the other specimens, although resembling in some respects that predicted recently by Maki for impure systems, is probably due to pinning effects. The results in lead are compared with those obtained in earlier work on indium and tin.

I. INTRODUCTION

Interest in the properties of the intermediate state in type-I superconductors has been revived by the work on vortex models of type-II superconductors.[1,2] The intermediate state[3] occurs when an arbitrarily shaped piece of superconducting material is placed in a uniform external magnetic field of magnitude smaller than the critical field H_c. The magnitude must be large enough so that owing to the diamagnetic distortion of the field lines the magnetic induction reaches H_c at some points inside the material. In an ellipsoidal specimen, the intermediate state occurs when the external field lies between $(1-n)H_c$ and H_c, where n is the demagnetizing coefficient of the ellipsoid. The specimen achieves thermodynamic equilibrium by dividing into a large number of superconducting and normal domains (typically of dimensions about 0.01 cm or smaller in directions perpendicular to the field). It is easy to show that in equilibrium the field in the normal regions is equal to H_c.

The existence of the intermediate state was first demonstrated by Meshkovsky and Shalnikov,[4] who found that the domain structure was very complicated. In spite of this, some of the early magnetization experiments were interpreted using models in which the specimen was assumed to break up into a regular array of alternately normal and superconducting laminae. This approach provided a reasonable quantitative description of the experimental results so long as the demagnetization factor was not close to unity. Other experiments, notably those using the "powder-pattern" technique,[5,6] have shown that the structure is not generally laminar although such a structure can be produced with the magnetic field inclined to the normal to the large faces,[6] by use of a rotating field,[7] or by passing a sufficiently strong current perpendicular to the magnetic field.[8] Another particularly simple structure was observed by DeSorbo[9] in very pure indium and by Träuble and Essman[10] in lead at fields much less than critical. The regions containing flux were found to be cylindrical with circular cross sections. Such regions are usually called flux tubes.

Studies of the structure of the intermediate state of lead have shown that the domains are long and meandering,[10] rather than laminar or cylindrical. However, because of the rather gross averages involved in the present experiments, the details of the flux distribution are not expected to be important.

Other experiments employing a wide variety of techniques have been carried out to examine, in addition to the structure, thermal and electrical transport properties of the intermediate state.[7,11] Partly because of its connections with flux motion in type-II superconductors,[1,2] interest has arisen recently in another nonequilibrium phenomenon, that of the propagation of displacement waves of the interphase boundary along the steady magnetic field direction.[12] Such waves can be induced by applying small-amplitude pulsed or oscillating magnetic fields to a specimen in the intermediate state. The method for studying the resulting wave motion is similar to that used in the study of helicon waves in metals[13,14] since, on a macroscopic scale, the average electromagnetic fields in the specimen resemble those in a medium through which a helicon wave propagates. In essence, the experiment involves setting up standing waves in a thin plate and measuring a resonant frequency and damping factor.

The choice of lead for the present experiments was made for a number of reasons. In order to obtain easily detectable resonances, one wants the highest possible value of the ratio RB/ρ, where R and ρ are the Hall coefficient and resistivity of the metal in its normal state and B is the magnetic induction (here, as we shall see, equal to H_c). A high value of this ratio corresponds to the cyclotron frequency of the carriers being large compared to the collision frequency. High-purity lead is readily available so that small values of ρ can be achieved at low temperatures. The relatively large critical field of lead also helps to increase the ratio RB/ρ. The high transition temperature is useful in making it easy to work at low reduced temperatures where the present theories are most likely to be applicable. By using single crystals of known orientation, it was also hoped that correlations between the intermediate-state properties and the anisotropic magnetoconductivity tensor of the normal state could be found.

II. THEORIES

There have been two different theoretical approaches to the low-frequency electrodynamic properties of the intermediate state. Nozières and Vinen[2] have given a macroscopic formulation of the hydrodynamics of the electron superfluid, taking into account its interaction with the normal region of a flux tube. A microscopic treatment has been given by Andreev[12]; it is based on the time-dependent Bogoliubov equations[14] which describe the excitations in a normal region in the vicinity of the boundary of the superconducting region.

Nozières and Vinen[2] consider a circular cylindrical normal region surrounded by superconducting material. The cylinder is assumed to contain magnetic flux of density H_c so that the material within the cylinder is in the normal phase. The radius of the flux tube is assumed to be much larger than the width of the interphase boundary layer which is of the order of the coherence length ξ and the penetration depth λ.[15] (We shall not be concerned here with cases where ξ and λ differ by as much as an order of magnitude.) A flux tube represents a single domain of the sort that might be found at the lowest fields at which the intermediate state could form.

The equation of motion of a flux tube, at $T = 0\,^\circ$K, is found by Nozières and Vinen,[2] from a generalization of the Magnus effect of classical hydrodynamics, to be

$$\vec{F} + Ne(\vec{v}_{s1} - \vec{v}_L) \times \vec{\phi}/c = 0 \ , \qquad (1)$$

where $\vec{F}$ is the external force on the electrons in the flux tube, N is the density of electrons in the flux tube, $\vec{\phi}$ is the magnetic flux in the tube, $\vec{v}_{s1}$ is the superfluid velocity outside the tube, and $\vec{v}_L$ is the velocity of the tube. (Gaussian units are used throughout.) By assuming mechanical equilibrium on the interphase boundary, it is shown that $\vec{v}_{Nc}$, the velocity of the normal electrons in the tube, is equal to $\vec{v}_{s1}$; i.e., the current is continuous across the core boundary. Assuming that the only external force on the (normal) electrons in the flux tube is due to lattice scattering, the equation of motion becomes

$$e(\vec{v}_{s1} - \vec{v}_L) \times \vec{H}_c/c - m\vec{v}_{s1}/\tau = 0 \ , \qquad (2)$$

where τ is the electronic relaxation time in the normal phase. A clear interpretation of Eq. (2) follows from rearranging it in the form

$$\vec{E} = \rho\vec{J} - R\vec{J} \times \vec{H}_c \ , \qquad (3)$$

where $\vec{J} = ne\vec{v}_{s1}$, $-\vec{v}_L \times \vec{H}_c/c = \vec{E}$ is the electric field induced in the flux tube by virtue of its motion, ρ is the resistivity, and R is the Hall coefficient in the normal region.

Formally, then, this approach yields the same constitutive equation as if the material were completely normal. When flux tubes (normal regions) move, currents must flow through the normal regions; the superconducting regions do not provide zero-resistance paths as they do if the normal regions are stationary. Recall, however, that Eq. (3) is derived for an isolated flux tube, that is, a single normal domain in a superfluid background. Hence, Eq. (3) describes fields and currents on the scale of a flux-tube diameter. In order to calculate the response of a large specimen, Eq. (3) must be averaged in some appropriate manner. If the flux-tube boundaries are very sharp, then a given volume of freely moving flux tubes contains the same amount of moving flux as a similar volume of normal material without superconducting boundaries. The specimen response is then determined by the fraction of the material that is normal. This description should be valid well below H_c where the normal regions are well isolated. It may also appear near H_c, even though isolated normal regions no longer exist, because the influence of superconducting boundaries becomes less important.

Equation (3) is just the constitutive equation on which the Chambers and Jones[16] theory for helicon waves is based. Adding the assumption of small-amplitude waves traveling along the axis of the flux tube and taking a time dependence of $e^{-i\omega t}$, we obtain the same dispersion formula as for helicon wave propagation, namely,

$$\omega = 2\pi f = k^2c^2(\rho_{xy} - i\rho_{xx})/4\pi \ , \qquad (4)$$

where $\rho_{xy} = RH_c$ and $\rho_{xx} = \rho$ depend only on the properties of the material in the normal regions and not on the relative sizes of these regions. In a symmetrically excited thin plate of thickness $2a$, the resonant frequencies are obtained by inserting $k = n\pi/2a$ (n = odd integer) into Eq. (4). The fundamental resonance ($n=1$), is then given by

$$f_R = c^2(\rho_{xy}^2 + \rho_{xx}^2)^{1/2}/32a^2 \ . \tag{5}$$

In the experimental arrangement used here, the magnetic field was kept fixed and the frequency of the waves was varied continuously through the resonance. We thus obtained f_R and Δf, the width of the resonance at half-maximum of the in-phase response. ρ_{xx} and ρ_{xy} were found from the relations derived by Chambers and Jones,

$$Q = f_R/\Delta f \ , \tag{6}$$

$$u = (4Q^2 - 1)^{1/2} = \rho_{xy}/\rho_{xx} \ , \tag{7}$$

$$\rho_{xx} = 16a^2\,\Delta f/c^2 \ . \tag{8}$$

These formulas are only valid for an infinite thin plate. However, corrections for specimens of finite size can be calculated from the work of Légendy.[17] Although the corrections may be many percent, they have a negligible influence on the field and temperature dependences reported here.

The work of Andreev[12] leads to results similar to those of Nozières and Vinen.[2] By averaging the microscopic electromagnetic fields over regions larger than the scale of the domain structure, he obtained the following macroscopic equations for $T = 0\,^\circ$K:

$$\begin{aligned} &\mathrm{div}\vec{B} = 0, \quad \mathrm{curl}\vec{E} = \frac{\partial\vec{B}}{c\,\partial t}, \quad (\vec{H}\cdot\vec{\nabla})\vec{H} = 4\pi\vec{J}_1\times\vec{H}/c \ , \\ &\vec{E}\cdot\vec{H} = 0, \quad |\vec{H}| = H_c \ , \quad \vec{B} = \gamma\vec{H} \ , \end{aligned} \tag{9}$$

where $\vec{J}_1$ is the projection of the current density in a plane perpendicular to the static field and γ is the fraction of normal material.

Equations (9) can be used to derive a wave equation whose dispersion formula is the same as the helicon dispersion law. The solution given by Andreev[12] for an infinite plate turns out to be almost identical to the normal-state helicon results, the only difference being that the amplitude of the resonance is proportional to γ.

Although both the Nozières-Vinen and Andreev theories have been developed for $T = 0\,^\circ$K, they should remain valid at higher temperatures so long as (i) the interphase boundary remains as "sharp" as has been assumed, and (ii) the quasiparticle density in the superconducting regions remains small. Above $T = 0\,^\circ$K, ρ and R should then be replaced by $\rho(H_c(T), T)$, $R(H_c(T), T)$.

In the present experiments, Q in the intermediate state and at fields just above H_c was no greater than 0.55. Taking Q to be constant, Eq. (6) reduces to

$$f_R = c^2 Q\rho/16a^2 \ , \tag{10}$$

so that

$$f_R(H_0, T)/f_R(H_c(T)) = \rho(H_0, T)/\rho(H_c(T), T) \ . \tag{11}$$

The voltage across the pickup coil at the maximum of the in-phase response is

$$V_P = \gamma[Q^2/(2Q+1)]H_c R\,G/4a^2 \ , \tag{12}$$

where G is a geometrical factor which can be determined experimentally. From this it follows that

$$V_P(H_0, T)/V_P(H_c(T)) = \gamma R(H_0, T)/R(H_c(T), T) \ , \tag{13}$$

where H_0 is the external field.

In considering the solution to the Bogoliubov equations, Andreev[12] found some novel features of quasiparticle reflection at the boundary between the superconducting and normal phases. If the temperature of the system is small compared with T_c, then only a negligible fraction of the excitations have energies greater than the energy gap Δ. Thus an electron impinging on the boundary from the normal side cannot pass through and exist as an excitation in the superconducting region. The electron forms a Cooper pair with another electron, thus leaving a holelike excitation in the normal region with momentum opposite to the momentum of the incident electron. The electron is thus reflected as a hole. The unusual aspect of this process is that all three components of the velocity of the excitation are reversed upon reflection; the quasihole moves away from the boundary along the incident electron trajectory. Thus, in a material whose electron mean free path is much greater than the dimensions of the normal regions, an excitation can oscillate back and forth between the interphase boundaries many times before being scattered. From these considerations, a size effect will be expected to occur when the thickness becomes comparable with the transverse dimensions of the normal regions and not with the electronic mean free path, as is the case in normal metals. A similar description of quasiparticle reflection has been given by McMillan.[17]

III. EXPERIMENTAL TECHNIQUES

The experimental techniques used in our two laboratories differed only slightly and will be designated C or R whenever it is necessary to refer to any feature not common to the two arrangements.

A. Specimen Preparation

Both polycrystalline [P] and single-crystal specimens were used in these experiments. All specimens were prepared from Cominco 69-grade lead. The polycrystalline specimens were pressed to the desired thickness between clean steel rollers and then etched to remove any surface contamination. The single-crystal specimens were cut from ingots either by electrical spark cutting (R) or by acid cutting (C) (using 80% glacial acetic acid and 20% hydrogen peroxide as the cutting fluid). The ingots were all grown using a modified Bridgman technique.[18] Chlorine-treated graphite (R) and pyrolytic boron nitride (C) were used for the crucibles. The lead did not adhere to the crucibles, so that straining on cooling was not a serious problem. From extrapolation to zero field of the resonant frequencies at 4.2 °K measured in fields greater than H_c (4.2 °K), resistance ratios of the single crystals grown in graphite were estimated to be about 13 000, and those grown in boron nitride in excess of 20 000. The ratio for the polycrystalline specimen was 8700. All specimens will be designated according to the orientation and laboratory where the experimental work was done; for example, [100]R refers to a single crystal whose axis normal to the largest faces of the specimen was along the [100] direction and which was run at Rutgers.

The orientations of the crystal axes of the ingots were found from back-reflection Laue photographs. In this experiment we wanted the large faces of the plate to be normal to the [110] axis because previous studies of the magnetoresistance had shown that this was a singular field direction in lead. (The damping of helicon waves is expected to be least if they propagate along such a direction.[19]) However, the main reason for using single rather than only polycrystalline specimens was to eliminate the scattering of electrons at the crystalline boundaries and so increase the mean free path. At such low fields as are required to produce the intermediate state, the collision frequency even in a good single crystal is expected to exceed the cyclotron frequency so that no sharp anisotropy near the singular field direction would be expected. For the purpose of comparison, crystals having plate normals along the [100] and [111] directions were also prepared.

Surfaces produced by the spark-erosion process have a roughness of the order of 10 μ. The heavily damaged layer was removed with a chemical etch and subsequent polish. Specimens prepared by acid cutting were given a light etch to smooth the surfaces. All specimens were in the form of approximately rectangular plates of area about 1 cm^2 and thickness about 0.2 mm (R) or in the range 0.65–0.90 mm (C). The specimen thickness varied as much as 3% in any given specimen.

B. Methods of Generation and Detection

The methods of generation and detection used here were similar to those used for helicon waves by Taylor, Merrill, and Bowers.[20] A pickup coil was wound closely around the specimen and then either surrounded by a single drive coil (R) or else placed between two separate drive coils (C) so that in either case a small-amplitude oscillating field was produced parallel to the specimen surfaces for the purpose of inducing the wave motion. The axis of the pickup coil was perpendicular to the drive-coil axis, thus minimizing direct coupling. A useful feature of the twin-drive coil specimen housing was that the pickup signal could be nulled in zero external field while the apparatus was immersed in the liquid helium. This was usually necessary for much of the small-signal data because the lack of electrical orthogonality can give an appreciable frequency-dependent signal having the same phase relationship to the reference signal as the phenomenon under study. Typical drive-field amplitudes were 0.5 G (R) and 2 G (C).

A general swept-frequency phase-sensitive detection system was used for these measurements. This feature was almost essential because one must study broad resonances in a limited field range, since the applied magnetic field must be less than the critical field H_c. In addition, in the normal state the resonance parameters change with magnetic field so it is more convenient to keep the field constant and sweep the frequency. For the purpose of these measurements, it was sufficient at fixed values of magnetic field to determine the frequency dependence of the component of the pickup voltage that was in phase with the excitation current. The output of the phase-sensitive detector was applied to the y axis of an x-y recorder, the x axis of which was driven by a voltage proportional to either the frequency or to the external field, depending on which of the two quantities was being varied.

C. Magnetic Field and Temperature Measurement

The static magnetic field was produced by either an electromagnet or a solenoid calibrated to an absolute accuracy of 1%. The field was kept constant to within 0.1% during any given set of swept-frequency measurements.

The detailed measurements between 1.2 and 4.2 °K were carried out with the specimen immersed in the liquid helium, the temperature of which could be accurately controlled to 0.01 °K by maintaining the vapor pressure above the liquid helium through a self-regulating diaphragm valve. In order to be able to raise the temperature of the specimen up to the superconducting transition at 7.2 °K, a double cannister enclosure was constructed. The specimen and coils were enclosed in the inner can around the outside of which was wound a heating coil. The larger can served to keep the liquid helium from contact with the inner can. The inside can and the space between the two cans contained helium gas maintained at pressures of 50 and 1 μ Hg, respectively. With these amounts of exchange gas, it took about 15 min for the temperature to reach equilibrium after a change in the heater current. The temperature of the specimen was measured with a three-lead germanium resistance thermometer to an accuracy of 1%.

D. Experimental Procedure

Figure 1 shows a typical set of resonance curves

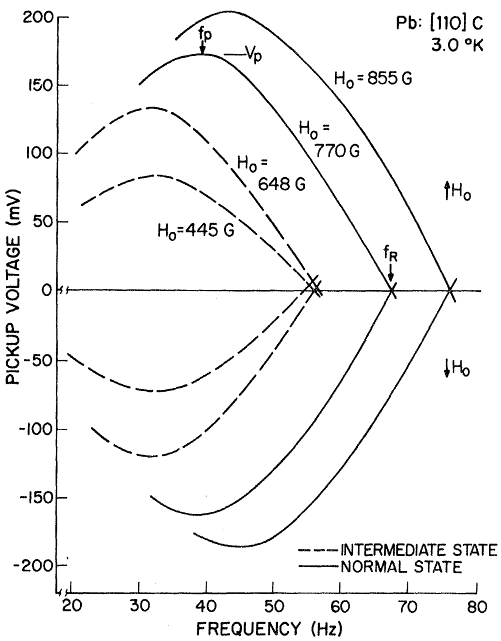

Fig. 1. Typical resonance curve.

for specimen [110]C. Data were taken for both normal and reverse field directions so that components of the pickup voltage even in the field could be subtracted out. Various parameters of a resonance curve are defined in Fig. 1. When the resonance curves were not symmetrical, the resonant frequency was taken to be the frequency where the normal and reversed field curves crossed. The peak frequency f_P and peak voltage V_P were obtained by averaging the values for the two field directions.

Because of hysteresis effects, the voltage obtained at a given static field depended on whether the field had been increased or decreased to reach that value. In order to obtain reproducible results, data were taken with the field always increasing in steps through the intermediate state. It was found that data obtained in this way were quite reproducible. Other consistent prescriptions also gave reproducible results, for example, when the magnetic field was reduced monotonically. However, although such data were reproducible, they were not exactly the same as those obtained by the first procedure using an increasing field. Nevertheless, the resonant frequencies did not depend upon the magnetic history. We believe that this reflects the fact that the normal regions contain an induction $B=H_c$ (as will be pointed out in Sec. IV E, this implies that the "pinning" effects were negligible). On the other hand, signal amplitudes did depend on the magnetic history, the effect being greatest at low fields. In a decreasing field, the signal voltage was larger than in an increasing field. This happens because any flux trapped in the specimen increases the average induction. The hysteresis patterns could be reproduced even without warming the specimen above T_c, thus indicating that the trapped flux in zero applied field (due primarily to pinning by defects) was quite small.

The demagnetization coefficient of an ellipsoid inscribed inside any specimen is very near unity so that one might expect that flux should start to enter the specimen at external fields very close to zero. This in turn would give $\gamma=B/B_c=H_0/B_c$. In fact, however, it was always found that flux did not enter until an external field of $H_P\simeq 0.3H_c$ was reached. This can be readily understood from considering the field configuration before flux penetration begins. The ends of the plate are flat and not nearly as sharp as the edges of the inscribed ellipsoid. If we assume the distortion of the field by a square plate to be approximately the same as that caused by a sphere of diameter equal to the length of the plate, then H_P should be given by the value of the external field for which the field at the equator of the sphere just reaches H_c, i.e., $H_P=\frac{1}{3}H_c$, close to the observed value for a wide range of thickness-to-width ratios. The data for $H_P<H_0<0.6H_c$ suggest that $\gamma\simeq(H_0-H_P)/(H_c-H_P)$, but that for $0.6\,H_c<H_0<H_c$, $\gamma\simeq H_0/H_c$.

IV. RESULTS AND DISCUSSION

A. Field Dependence of f_R

Figure 2 shows how the resonant frequency f_R at different temperatures depended on the external field H_0 for samples [110]C, [110]R, and [100]C. The results for [P]C and [P]R were qualitatively similar to those for [110]R and [100]C. At fields greater than $0.4\,H_c$, the asymmetry referred to in Sec. III D was less than 1% for specimen [110]C. The greatest asymmetry was observed in specimen [100]C, but even in this case it was less than 8%. Note that for most specimens, Fig. 2 shows that the resonant frequency at low temperature (~ 1.2 °K) increased as the external field was reduced below H_c but that the opposite was true at higher temperatures. In specimen [110]C the dependence on external field was quite small, as can be seen in Fig. 3 where all the data for that specimen are plotted in reduced units, i.e., $f_R(H_0, T)/f_R(H_c(T))$ vs $H_0/H_c(T)$. Note that the original experimental data may be recovered by referring to the table in the figure.

The results in Fig. 3 are very close to both the prediction of Andreev's theory (that f_R should be independent of H_0) and the results of the Nozières and Vinen flux-tube model (although with H_0 close to H_c the intermediate state no longer resembles independent flux tubes). As can be seen from Fig. 2, the agreement is less satisfactory in the other specimens. [110]R, because of its method of prep-

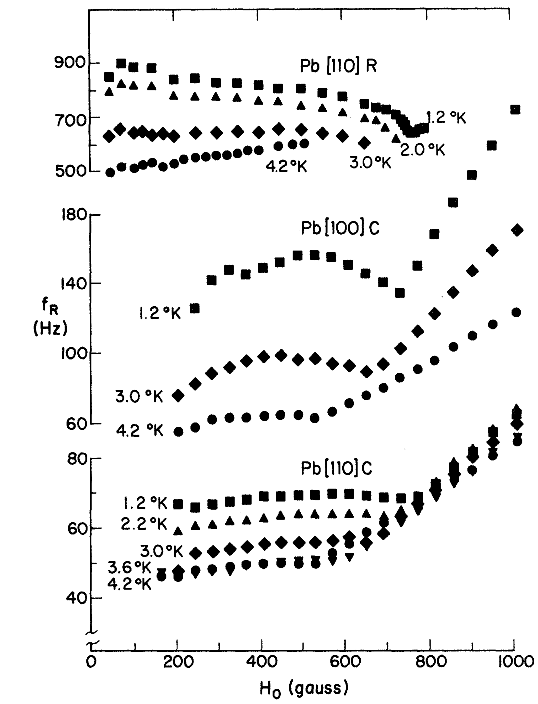

FIG. 2. Resonant frequency versus external magnetic field for samples at different temperatures. Data for $H_0 > H_c$ for [110]R not shown.

aration, was not as pure as [110]C and, in addition, was probably not as free from strain and other crystal damage. Possible causes of the change of resonant frequency with external field are discussed in Sec. IV F.

B. Temperature Dependence of f_R

The resonant frequency is related to the resistivity tensor elements by Eq. (5). In the normal state, ρ_{xx} and ρ_{xy} are functions of magnetic field and temperature. Experiments of Phillips[21] have shown that lead obeys Kohler's rule, namely,

$$[\rho_{xx}(B, T) - \rho_{xx}(0, T)]/\rho_{xx}(0, T) = \psi(B/\rho(0, T)) ,$$

where it is found that $\psi(w) = Cw^2$ and C is a constant. This implies that the temperature dependence of $\rho_{xx}(B \neq 0, T)$ is not of the same functional form as $\rho_{xx}(0, T)$; in fact,

$$\rho_{xx}(B, T) = CB^2/\rho_{xx}(0, T) + \rho_{xx}(0, T) . \qquad (14)$$

Borovik[22] has found that $\rho_{xx}(0, T) = \alpha + \beta T^4$. In the intermediate state, $B = H_c(T) = (1 - t^2)H_c(0)$ approximately, so that $\rho_{xx} = \rho_{xx}(H_c(T), T) = \rho_{xx}(T)$. For all resonances studied here, $Q < 0.55$ so that $\rho_{xy} < 0.5\rho_{xx}$. To a first approximation the resonant frequency is therefore governed by ρ_{xx}. Some allowance for the temperature dependence of ρ_{xy} can be made by supposing that the Hall coefficient R is independent of temperature; thus the temperature dependence of ρ_{xy} is given by $\rho_{xy} = R(1 - t^2)H_c(0)$.

At low temperatures, the second term in Eq. (14) can be neglected and $\rho_{xx}(B, T)$ should then be proportional to $(1 - t^2)$. For temperatures approaching T_c, the second term begins to increase rapidly, and $\rho_{xx}(B, T)$ will increase correspondingly. These predictions are confirmed by the data in Fig. 4. The initial decrease in f_R at low temperatures was found to be parabolic, and this was followed by a steep rise in f_R as T approached T_c.

That a qualitatively correct explanation of the temperature dependence of f_R can be obtained by considering the temperature dependence of only the normal regions will be true only so long as the number of quasiparticles excited in the superconducting regions is small. In lead, the minimum single-particle excitation energy Δ at $T = 0$ is 15 °K. Thus at 4.2 °K, the probability of occupation of an excited state just above the gap is only $e^{-3.6}$.

Data taken under conditions different from those applying in Fig. 4 showed the same general characteristics. Owing to heavy damping, it was not

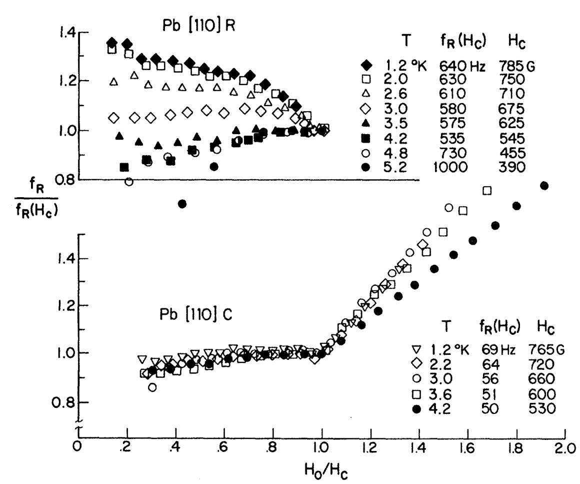

FIG. 3. Data at different external fields and temperatures for [110]C and [110]R plotted in reduced units.

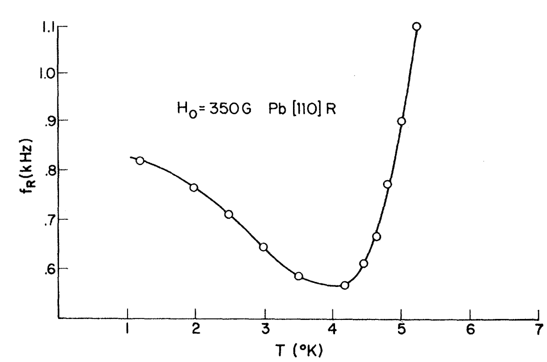

FIG. 4. Resonant frequency versus temperature.

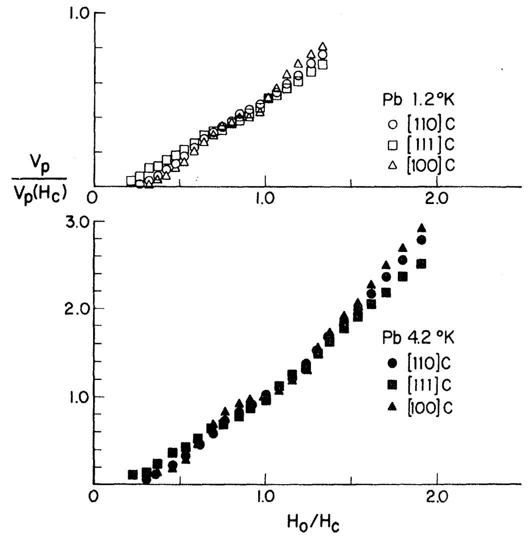

FIG. 5. Peak in-phase secondary voltage versus external magnetic field, for [110]C, [111] C, and [100]C.

possible to obtain results above about 5.25 °K.

C. Field and Temperature Dependence of V_P

The field dependence of the pickup voltage $V_P(H_0)$ for specimens [110]C, [111]C, and [100]C is shown in Figs. 5 and 6. For convenient comparison we plot $V_P(H_0, T)/V_P(H_c(T))$ against $H_0/H_c(T)$. It can be seen in Fig. 6 that all data for specimen [110]C fall on the same curve, which is close to a straight line of slope $H_c(T)/[H_c(T)-H_P(T)]$. It appears as though $H_P(T)/H_c(T)$ is independent of T and that the amount of flux, in units of $H_c(T)$, is independent of T for a given value of $H_0/H_c(T)$. Such results might be expected for an ellipsoid of well-defined demagnetization coefficient, but is surprising to find such behavior in a rectangular parallelepiped. However, it can be seen that the scaling between different specimens is not as good as the scaling with temperature of any given specimen. The discrepancies are greatest for [100]C, which, in addition, does not have a linear field dependence near H_c. These discrepancies, probably of the same origin as the field dependence of f_R, are discussed in Sec. IV F.

D. Hall Angle

In high-Q helicon resonances, the Hall angle can be obtained from direct measurements of Q. This was not practicable in the present experiments, where $Q\simeq 0.55$, because a small error in Q produces a large error in $u(H_0)$. However, by using the obvious relation

$$u(H_0)/u(H_c)=V_P(H_0)f_R(H_c)/\gamma V_P(H_c)f_R(H_0) ,$$

it is possible to obtain an estimate of the Hall angle in the normal regions. Both $f_R(H_0)/f_R(H_c)$ and $V_P(H_c)$ were obtained from direct measurement, while γ was determined from the variation of $V_P(H_0)/V_P(H_c)$ with H_0/H_c. The accuracy of using this procedure to determine $u(H_0)$ is limited by the difficulty in obtaining an accurate estimate of $u(H_c)\sim 0.1$.

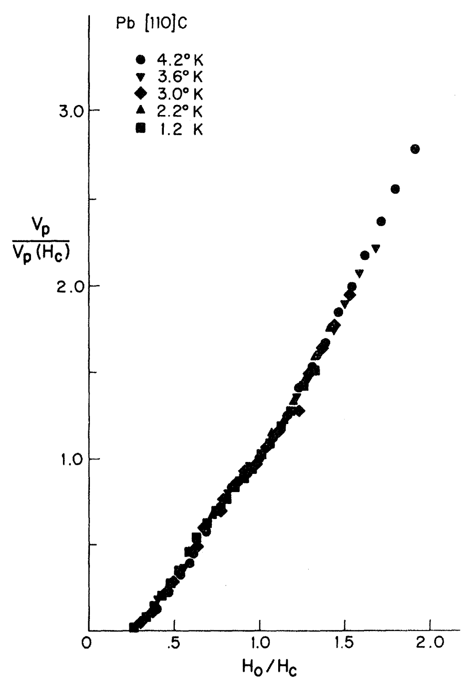

FIG. 6. Peak in-phase secondary voltage versus external magnetic field for [110]C at different temperatures in reduced units.

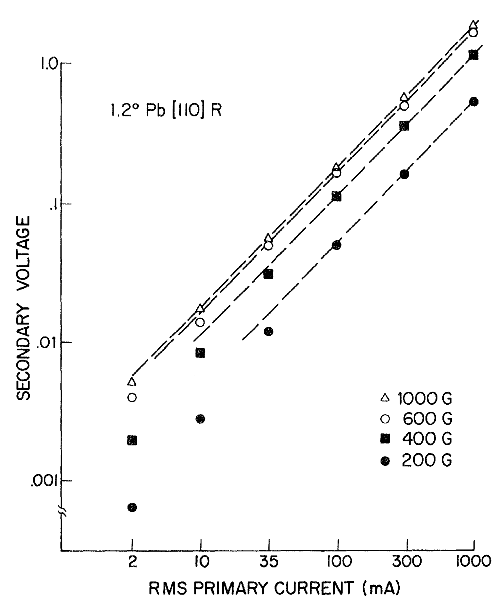

Fig. 7. In-phase secondary voltage versus primary current amplitude.

E. Dependence on Amplitude of Exciting Field

Because the theories described in Sec. II all produce a linear constitutive equation, it is expected that the amplitude of the secondary coil voltage should vary linearly with the amplitude of the current in the primary coil. It can be seen from Fig. 7, however, that for a small primary current this is not the case. Experience with flux flow in the mixed state of type-II superconductors suggests that the nonlinearity is due to "pinning" of the phase boundary. This prevents some of the flux within the specimen from responding completely to the ac field. Pinning phenomena are well known in type-II superconductors,[23] where vortex lines become trapped in potential wells caused by inhomogeneities. The passage of a direct current through the vortex array in the mixed state does not result in flux motion unless the current is strong enough to overcome the pinning forces. The situation with an alternating current is slightly different. Even though the amplitude of the current is less than the minimum direct current required to overcome pinning, the vortices may still be able to undergo oscillations within their potential wells (although the amplitude may be smaller than if the vortices were free).[24] Pinning effects in direct current flow have been observed in the intermediate state of polycrystalline lead by Chandrasekhar *et al.*[25] Further possible effects of pinning in the present experiments are discussed in Sec. IV F.

F. Deviations from Theory

The dependence of the resonant frequency on external field in the intermediate states of specimens [100]R, [100]C, P[C], and [P]R may be analogous to the effect in impure type-II superconductors recently discussed by Maki,[26] except that the variation of the effect with temperature is opposite from what would be expected from that theory. However, it is also possible that the field dependence arises from pinning of the phase boundary by inhomogeneities.

In the intermediate state, the effect of pinning on a moving phase boundary is to enable the magnetic field in the normal region to exceed the critical value. The potential barrier adds an additional force to the boundary so that the normal region is no longer in equilibrium with the adjacent superconducting region. An increase in the field raises the magnetoresistance of the normal material to $\rho(H', T)$, where $H' > H_c$. The resonant frequency, which is approximately proportional to $\rho(H', T)$ is therefore raised above the value expected for $\rho(H_c(T), T)$. At the lowest temperatures, the resonant frequency passes through a maximum at $H_0 \simeq \frac{1}{2} H_c$ because at such fields the interphase boundary area is a maximum (and hence pinning effects will be largest). As the temperature is raised, the phase boundary becomes less sharply defined and the pinning forces correspondingly smaller.

As mentioned earlier, comparison of the measured secondary voltage amplitude V_P with theory depends strongly on the demagnetization effects; no obvious discrepancy exists here.

G. Comparison with Tin and Indium

Figure 8 shows typical results obtained in indium and tin in earlier work.[27] As can be seen, the behavior in tin follows closely the predictions of theory. The deviations in indium are probably due to the size effect expected from Andreev's treatment of the scattering of excitations at the interphase boundary. In the normal state there is a large contribution to the resistivity from scattering at the surface of the specimen. However, once the intermediate state forms and the transverse dimensions of the normal regions become smaller than the plate thickness, most of the excitations rarely encounter the surface and so the effective resistivity drops to the bulk resistivity. Hence, the resonant frequency falls sharply.

The apparent absence of the pinning effects in indium and tin in contrast to lead has been previously noted by DeSorbo[9] in connection with his optical studies of the intermediate state. Pinning is favored in lead by the relatively small coherence length and the tendency (observed experimentally) for the domain structure to be more finely divided than in either indium or tin.

H. Conclusions

The dependence of the resonant frequency of heli-

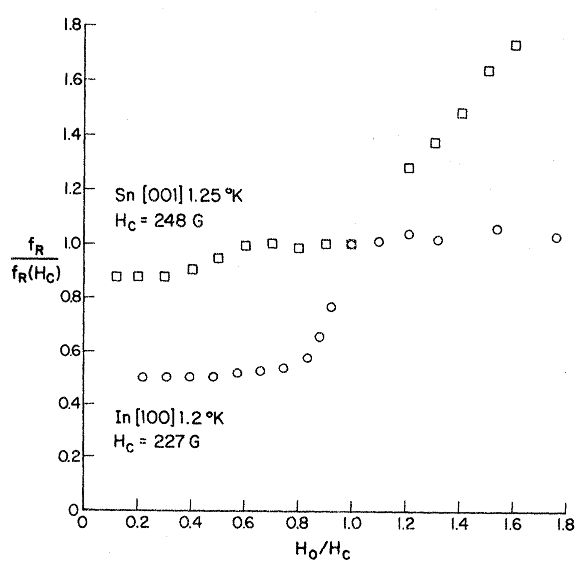

FIG. 8. Resonant frequency versus external magnetic field for indium (Kushnir, Ref. 27) and tin (Hays, Ref. 27).

conlike standing waves in the intermediate state on both external field and temperature appears to be well accounted for in good quality pure specimens by the microscopic theory of Andreev[12] and also by the model of Nozières and Vinen.[2]

Deviations from the theory in the lower-quality specimens appear to follow the trends attributed in earlier work to pinning of the interphase boundary by inhomogeneities rather than being explainable in terms of the impurity effect recently discussed by Maki.[26]

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